Northern Beaches Secondary College
Manly Campus

2019 HIGHER SCHOOL CERTIFICATE EXAMINATION

Trial Examination

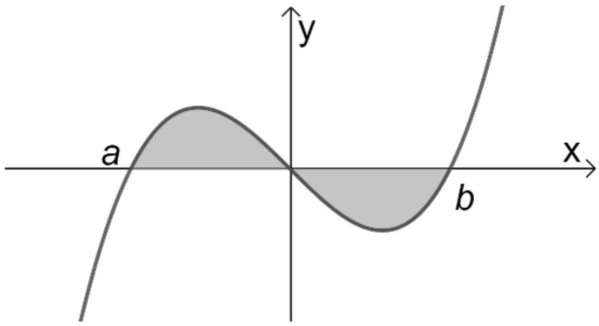
Mathematics

General Instructions	• Reading time – 5 minutes • Working time – 3 hours • Write using black pen • NESA approved calculators may be used • A reference sheet is provided • In Questions 11 – 16, show relevant mathematical reasoning and/ or calculations
Total marks: 100	**Section I – 10 marks** (pages 2-6) • Attempt Questions 1 – 10 • Allow about 15 minutes for this section **Section II – 90 marks** (pages 7-17) • Attempt Questions 11 – 16 • Allow about 2 hours and 45 minutes for this section

Section I - 10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1 The graph of a function $y = f(x)$, where $f(-x) = -f(x)$ is shown below.
The graph intersects the x-axis at $x = a, x = 0$ and $x = b$.



Which one of the below gives the area of the shaded region?

A. $\int_a^b f(x)\ dx$

B. $2\int_a^{a+b} f(x)\ dx$

C. $2\int_0^{-a} f(x)\ dx$

D. $-2\int_b^0 f(x)\ dx$

2 The gradient of the normal to the curve $f(x) = 3x^3 - 4x + 2$ at the point (-1, 3) is:

A. 5

B. -5

C. $-\frac{1}{5}$

D. $-\frac{1}{3}$

3 It is known that for a particular quadratic function, $\alpha + \beta = -\frac{5}{3}$ and $\alpha\beta = \frac{7}{3}$. The quadratic function could be:

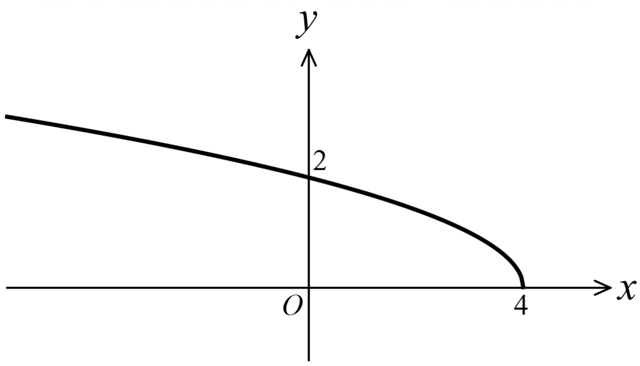
A. $f(x) = 3x^2 - 5x + 7$

B. $f(x) = 3x^2 + 5x + 7$

C. $f(x) = 3x^2 + 5x - 7$

D. $f(x) = 5x^2 - 7x + 3$

4 The graph of the curve $y = \sqrt{4 - x}$ is shown below.



For what value(s) of x is the curve not differentiable?

A. $x < 4$

B. $x = 4$

C. $0 < x < 4$

D. $x = 0$

5 Consider the points A (1, -2) and B (3, 6). What is the equation of the perpendicular bisector of AB?

A. $y - 2 = -\frac{1}{4}(x - 2)$

B. $y - 2 = 4(x - 2)$

C. $y - 4 = -1(x - 1)$

D. $y + 2 = -\frac{1}{4}(x - 1)$

6 It is given that $\ln a = 2\ln b + \ln c - \ln d$. Which of the following statements is true?

A. $\ln a = \ln\left(2b + \frac{c}{d}\right)$

B. $\ln a = \frac{cb^2}{d}$

C. $a = \frac{b^2 c}{d}$

D. $\ln a = \frac{\ln(b^2 + c)}{\ln d}$

7 If $\int_0^2 f(x)\,dx = 6$ what is the value of $\int_0^2 [x - 2f(x)]\,dx$?

A. -10

B. -5

C. 8

D. 12

8 Which of the following possible *k* values will there be two distinct rational solutions for the equation.

$2x^2 + 20x + k = 0$

A. $k = 52$

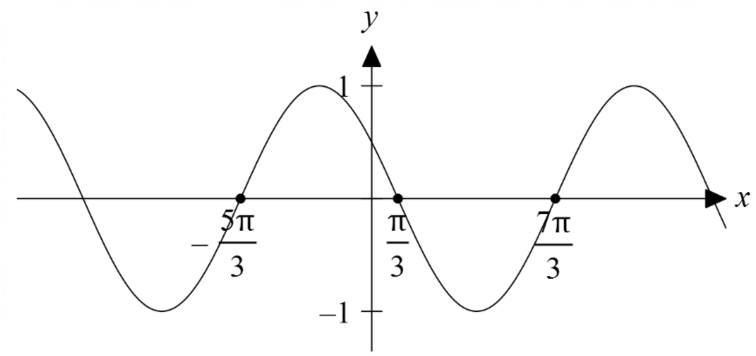
B. $k = 50$

C. $k = 48$

D. $k = 46$

9 Which diagram shows the graph $y = \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)$?

A.



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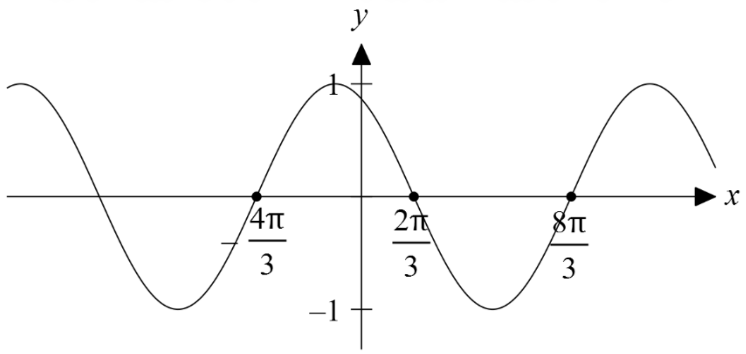

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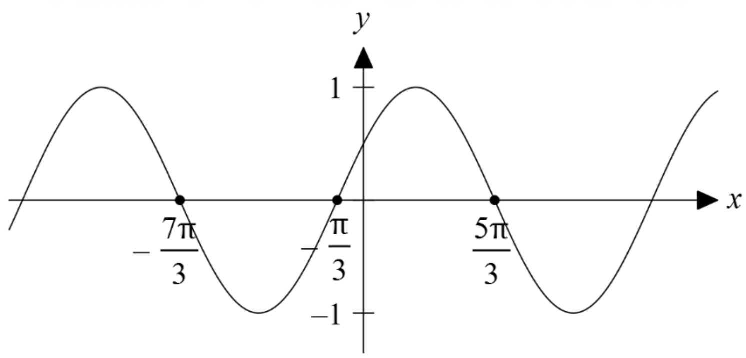
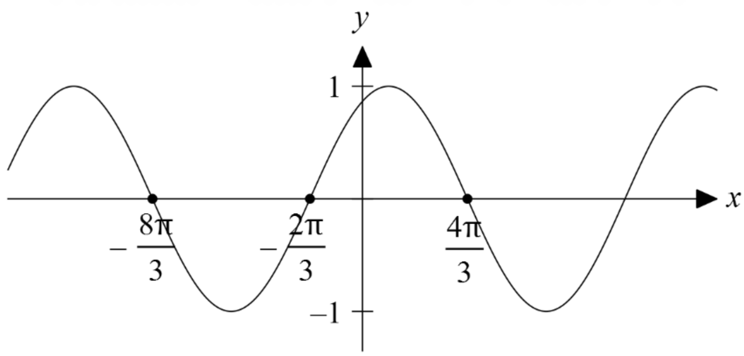


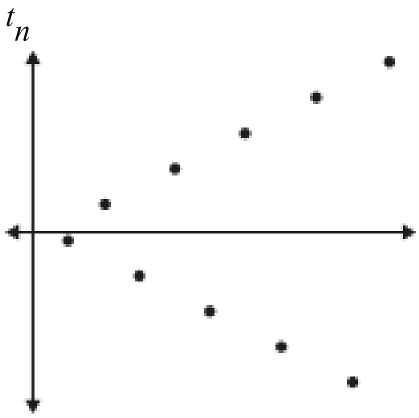
D.



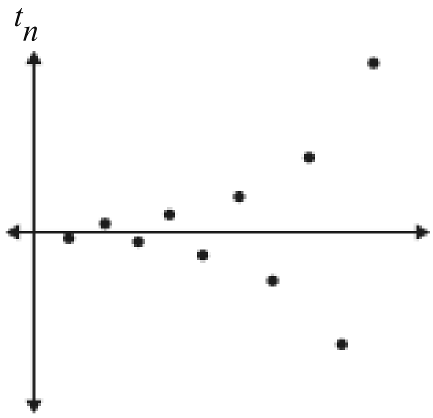
10 The first term of a geometric sequence is a, where $a < 0$. The common ratio of this sequence, r, is such that $r < -1$.

Which one of the following graphs best shows the first 10 terms of this sequence?

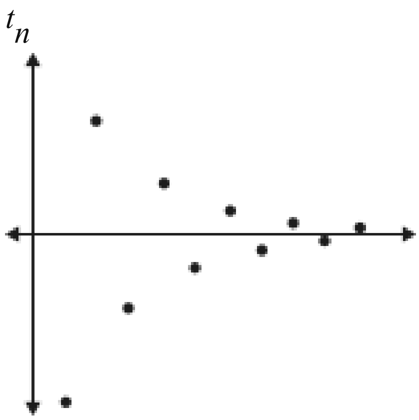
A.



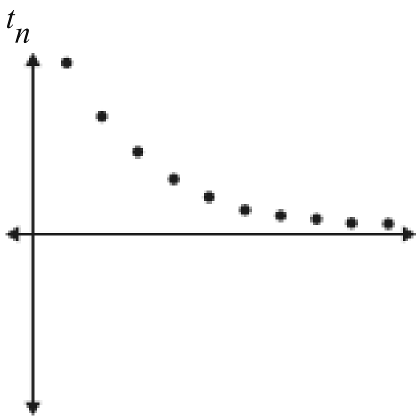
B.



C.



D.



End of Multiple Choice

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your response should include relevant mathematical reasoning and/ or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet — Marks

(a) Evaluate $\int_1^5 (3x - 7)\ dx$ — **2**

(b) Solve $|2x - 5| < 7$ — **2**

(c) Find the limiting sum of the geometric series, $2 - \frac{1}{3} + \frac{1}{18} - \frac{1}{108} + \ldots$. — **2**

(d) Differentiate with respect to x

i. $\frac{x^2}{e^{2x}}$ — **2**

ii. $\cos^3(5x + 3)$. — **2**

(e) Determine $\int \frac{3dx}{(3 - 2x)^3}$ — **2**

(f) Find the coordinates of the focus of the parabola $y = -\frac{1}{8}x^2 + x - 1$. — **3**

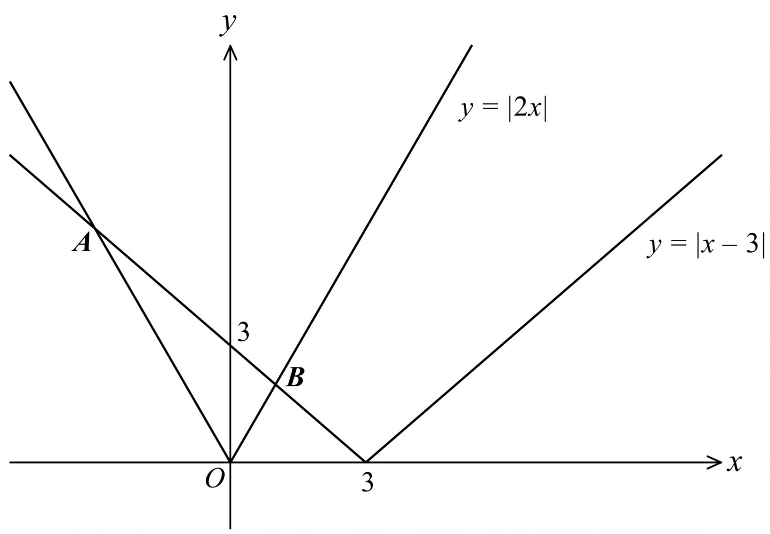
End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet　　Marks

(a) Find the point on the graph of $f(x) = x^2 - 5x + 4$ where the tangent has a gradient of -3. **2**

(b) Given $y = x\sqrt{1-x^2}$, show is $\frac{dy}{dx} = \frac{1-2x^2}{\sqrt{1-x^2}}$ **3**

(c) The diagram shows the graphs of $y = |2x|$ and $y = |x-3|$ which intersect at points A and B.



(i) Find the x – coordinate of points A and B. **2**

(ii) Hence or otherwise, solve $|x-3| \leq |2x|$ **1**

(d) Given $y = \ln(\cos x)$

(i) Differentiate $\ln(\cos x)$ with respect to x. **1**

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \tan x \; dx$ **2**

Question 12 continues on next page

(e) The temperature T, in degrees Celsius, of an oven is given by the equation

$$T = 120 + 80\sin\left(\frac{\pi}{3}t\right) \text{ where } t \text{ is measured in hours.}$$

(i) What is the range of temperature in the oven? **1**

(ii) What is the rate of change of the temperature when the oven first reaches two thirds of its initial temperature? Leave your answer in exact form. **3**

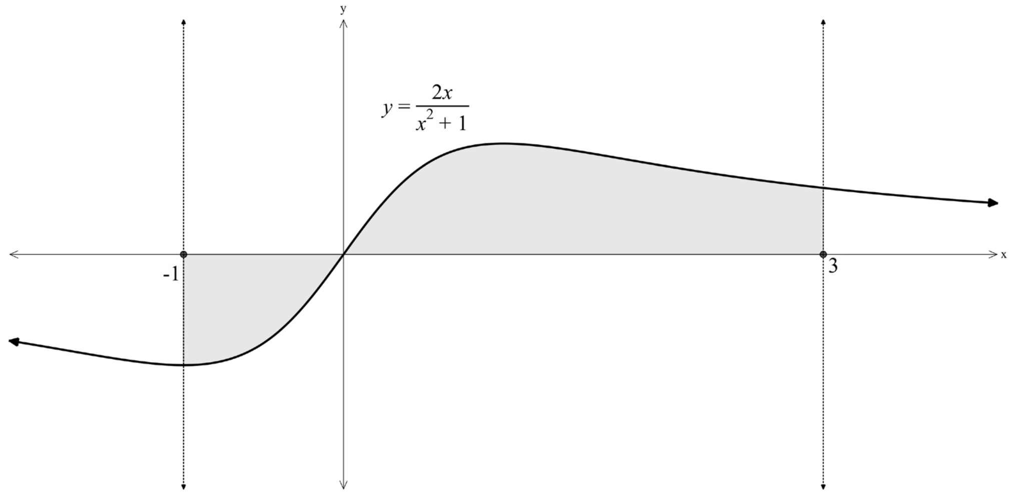
End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet | Marks

(a) Prove the following $\dfrac{1-\tan^2 x}{1+\tan^2 x} = 1 - 2\sin^2 x$ **3**

(b) The diagram shows the graph of $f(x) = \dfrac{2x}{x^2+1}$ The shaded area is

enclosed by the curve $f(x) = \dfrac{2x}{x^2+1}$, the x-axis, $x = -1$ and $x = 3$



Use Simpson's Rule and the 5 function values to approximate the shaded area.

Give the answer correct to 1 decimal place. **3**

(c) Gallium-67 is a radioactive element that decays over time.

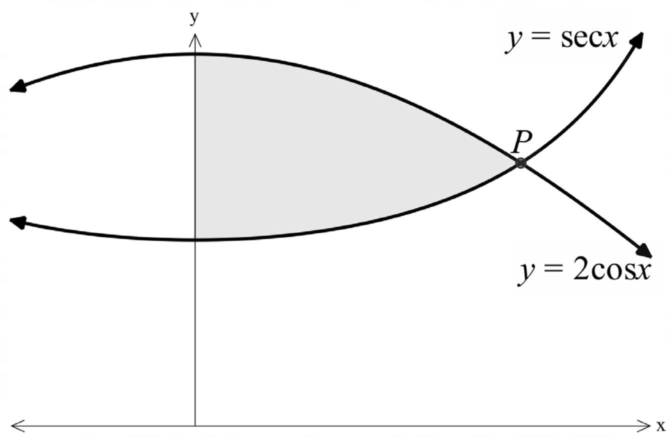
The mass of gallium-67 after t hours is given by $M(t) = Ae^{kt}$.

i. Show that $M(t)$ satisfies $\dfrac{dM}{dt} = kM$ **1**

ii. Given that the half-life of gallium-67 is 80 hours, find the value of k to 3 significant figures. **2**

iii. How many hours will it take for a mass of gallium-67 to reach 10% of its original mass? **2**

(d) The diagram below shows the shaded area bounded by the curves $y = \sec x$, $y = 2\cos x$ and the y – axis.



i. Show that the x-coordinate of P is $x = \dfrac{\pi}{4}$ **1**

ii. Given that $\cos^2 x = \dfrac{1}{2}(\cos 2x + 1)$, determine the volume of the solid of revolution produced by rotating the shaded area around the x-axis. **3**
(Give your answer in exact form)

End of Question 13

Question 14 (15 Marks) Use Q14 Answer Booklet. Marks

(a) Consider the function $f(x) = x(x-2)^4$

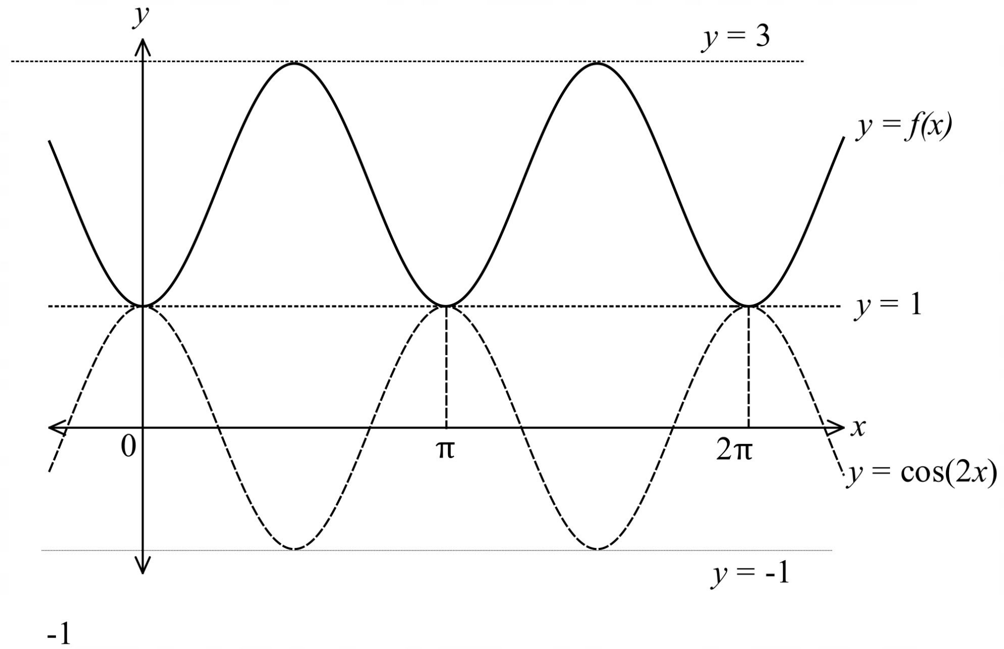
(i) Show that $f'(x) = (x-2)^3(5x-2)$. **1**

(ii) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. **2**

(iii) Sketch the graph of $y = f(x)$, showing the intercepts and stationary points. **2**

(b) The diagram shows the graphs of $y = \cos(2x)$ and $y = f(x)$ from $x = 0$ to $x = 2\pi$.

The graph of $y = f(x)$ is a reflection of $y = \cos 2x$ along the line $y = 1$.

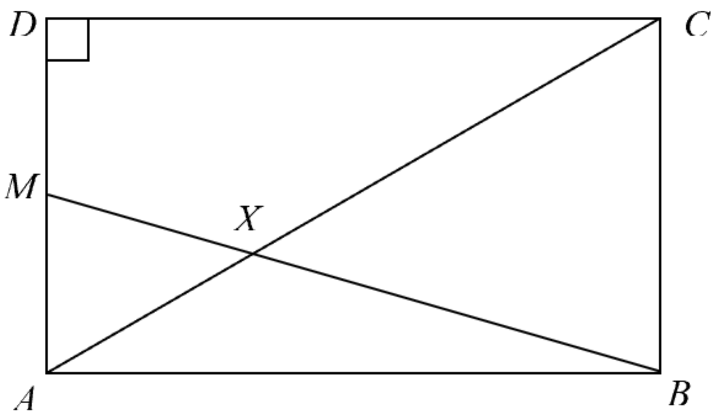


(i) Determine the equation for the graph of $y = f(x)$ **2**

(ii) Find the exact area between the curves $y = \cos(2x)$ and $y = f(x)$ from $x = 0$ and $x = 2\pi$ **3**

Question 14 continues on the next page

(c) In the diagram, $ABCD$ is a rectangle and $AB = 2AD$. The point M is the midpoint of AD. The line BM meets AC at X.



NOT TO SCALE

(i) Prove that the triangle AXM and CXB are similar. **2**

(ii) Hence show that $3CX = 2AC$. **1**

(iii) Show that $9(CX)^2 = 5(AB)^2$ **2**

End of Question 14

Question 15 (15 Marks) Use Q14 Answer Booklet Marks

(a) Show $\displaystyle\int_0^{\sqrt{e}} \frac{x^2}{x^3 + e}\, dx = \frac{\ln(\sqrt{e} + 1)}{3}$ **3**

(b) Jacqueline is a Year 12 student who plans to travel after completing the HSC exams.

On the 1st January 2019, Jacqueline has \$4000 in her savings account however she aims to have a **total** of \$7000 in her account by the 1st December 2019.

She will make monthly deposits of \$*M* at the **end** of every month with the first deposit to be made on the 31st January 2019 and the last deposit to be made on the 30th November 2019.

The bank pays 6% p.a. interest compounded monthly.

(i) How much will the \$4000 in Jacqueline's account accumulate to by 1st December 2019? Give the answer correct to the nearest cent. **1**

(ii) Calculate Jacqueline's monthly deposit, \$*M*, so that she has a **total** of \$7000 in her savings account by 1st December 2019. Give the answer correct to the nearest cent. **3**

Question 15 continues on the next page

(c) In the diagram the curve $y = 4x^3 - 9x^2$ and the line $y = -5x$ intersect at points origin O, C and D. Point A lies on the curve $y = 4x^3 - 9x^2$ between points O and C.

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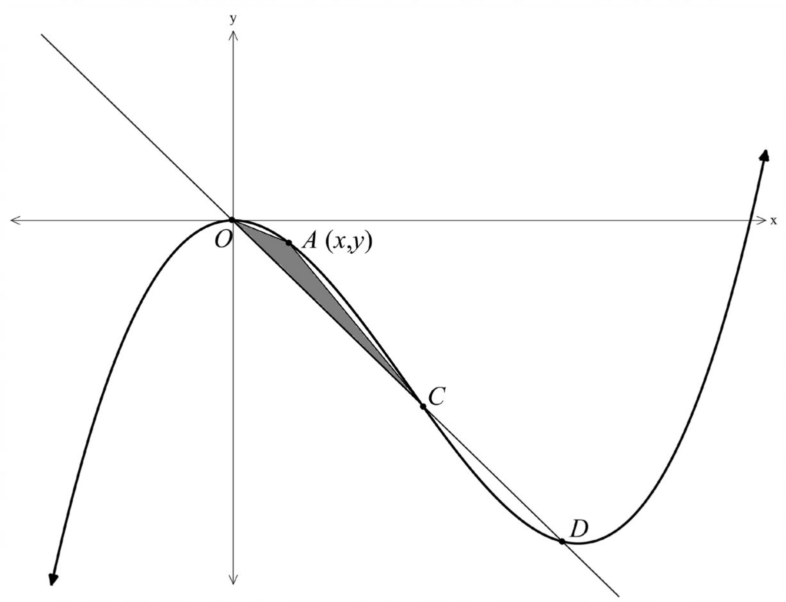


(i) Determine the coordinates of C. **2**

(ii) Show that the perpendicular distance between point A and the line segment OC is $\frac{1}{\sqrt{26}}\left(4x^3 - 9x^2 + 5x\right)$. **2**

(iii) Show that the area of the triangle AOC is $2x^3 - \frac{9}{2}x^2 + \frac{5}{2}x$. **1**

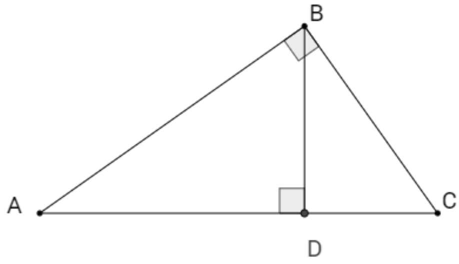
(iv) The point A is chosen so that the area of the triangle AOC is a maximum.

Find the maximum possible area correct to two decimal places. **3**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet Marks

(a) In the diagram, triangle ABC is given such that $AB \perp BC$ and $BD \perp AC$.



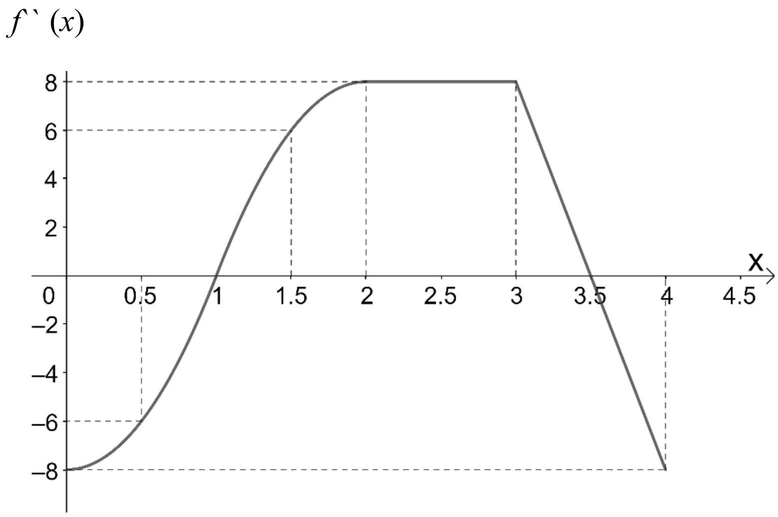
i. By stating an appropriate triangle similarity, show that $AB^2 = AC.AD$. **1**
(Note: You do **NOT** have to prove similarity)

ii. Hence, or otherwise, show that $AB^2 + BD^2 = AD(AD + 2CD)$. **2**

(b) Let $f(x)$ be a function defined for $0 \leq x \leq 4$ such that $f(0) = 0$.

The diagram shows the graph of its derivative, $y = f'(x)$.

It is known that $\int_0^2 f'(x)\ dx = 0$.



(i) For which values of x is $f(x)$ decreasing? **1**

(ii) Find $f(4)$. **1**

(iii) What is the maximum value of $f(x)$? **1**

(iv) Draw a graph of $y = f(x)$ for $0 \leq x \leq 4$. **2**

Question 16 continues on the next page

(c) Beatrice takes out a home loan of $520,000.

The loan is charged reducible interest of 8.4% per annum, calculated monthly.

The loan is to be repaid in equal monthly repayments of $\$M$ over 15 years.

Let A_n be the amount owing after the *n*th repayment.

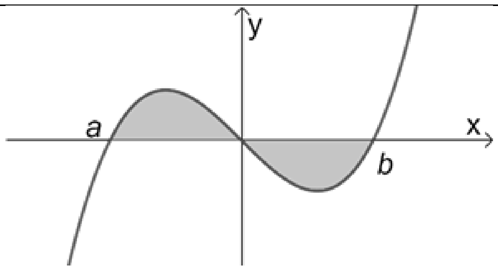
(i) **Derive** an expression for A_3, the amount owing after 3 months. **2**

(ii) Show that the monthly repayment is approximately $5090.21. **2**

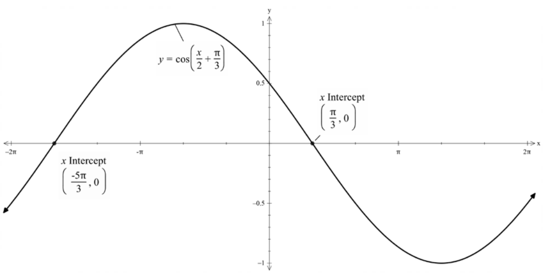
(iii) Immediately after her 24^{th} payment, Beatrice makes a one-off payment of $20,000. If the interest rate and monthly repayment remain unchanged, after how many more months will Beatrice pay off the loan? **3**

End of Examination

Multiple Choice

Q1	 As $f(x)$ is odd then 1. $b = -a$ 2. Let $\int_a^0 f(x)\,dx = k$ $\therefore \int_0^b f(x)\,dx = -k$ 3. Area = $k + \lvert -k \rvert = 2k$ -- A. $\int_a^b f(x)\,dx = k + (-k) = 0$ -- B. $2\int_a^{a+b} f(x)dx$ $= 2\int_a^{a-a} f(x)dx$ $= 2\int_a^0 f(x)dx$ $= 2k$ – CORRECT -- C. $-2\int_b^0 f(x)\,dx$ $= -2 \times -\int_0^b f(x)\,dx$ $= -2 \times -1 \times -k$ $= -2k$ -- D $2\int_0^{-a} f(x)\,dx$ $= 2\int_0^b f(x)\,dx$ $= 2 \times -k$ $= -2k$	B

Q2	$f(x) = 3x^3 - 4x + 2$ $f'(x) = 9x^2 - 4$ $f'(-1) = 9 - 4$ $= 5$ Therefore gradient of normal is $-\frac{1}{5}$	C
Q3	$f(x) = 3x^2 + 5x + 7$	B
Q4	From the graph the tangent is vertical where x = 4	B
Q5	$m = -\frac{3-1}{6+2} = -\frac{1}{4}$ $\text{midpoint} = \frac{1+3}{2}, \frac{-2+6}{2}$ $= (2,2)$ $y - 2 = -\frac{1}{4}(x-2)$	A
Q6	$\ln a = \ln b^2 + \ln c - \ln d$ $= \ln\left[\frac{b^2 c}{d}\right]$ $\therefore \quad a = \frac{b^2 c}{d}$	C
Q7	$\int_0^2 x - 2f(x)\ dx$ $= \int_0^2 x dx - 2\int_0^2 f(x) dx$ $= \left[\frac{x^2}{2}\right]_0^2 - 2(6)$ $= \left(\frac{2^2}{2} - \frac{0^2}{2}\right) - 2(6)$ $= -10$	A

Q8	$2x^2 + 20x + k = 0$ Distinct means $\Delta > 0$ Rational means Δ is a perfect square. $\Delta = b^2 - 4ac$ $= 20^2 - 4(2)(k)$ $= 400 - 8k$ If $k = 48$, then $\Delta = 16$, which is both positive and a perfect square.	C
Q9	$y = \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)$ $0 \le \frac{x}{2} + \frac{\pi}{3} \le 2\pi$ $-\frac{\pi}{3} \le \frac{x}{2} \le \frac{5\pi}{3}$ $-\frac{2\pi}{3} < x \le \frac{10\pi}{3}$ Shift left of $\frac{2\pi}{3}$ 	A
Q10	Let r be an integer ∴ $r \Rightarrow r^2 \Rightarrow r^3 \Rightarrow r^2 \Rightarrow r^5$ $-2 \Rightarrow 4 \Rightarrow -8 \Rightarrow 16 \Rightarrow -32$ Therefore Option B	B

Question 11

11a	$\int_1^5 (3x-7)\ dx$ $= \left[\frac{3x^2}{2} - 7x\right]_{,1}^{5}$ $= \frac{75}{2} - 35 - \left(\frac{3}{2} - 7\right)$ $= 8$	2 marks correct answer 1 mark correct integration **Note:** this does not equate to an area so no units
b	$\|2x - 5\| < 7$ $-7 < 2x - 5 < 7$ $-2 < 2x < 12$ $-1 < x < 6$	2 marks correct answer 1 mark correct boundary values **Note:** stating separate inequalities requires an 'and' not an 'or' 6>x>-1 is never acceptable!
c	$2 - \frac{1}{3} + \frac{1}{18} - \frac{1}{108} + \ldots$ $S_\infty = \frac{2}{1 - -\frac{1}{6}}$ $= \frac{12}{7}$	2 marks correct solution 1 mark correct ratio or cfe - carry forward error
d i	$\frac{x^2}{e^{2x}}$ $u = x^2$ $u' = 2x$ $v = e^{-2x}$ $v' = -2e^{-2x}$ $y' = 2xe^{-2x} + \ -2e^{-2x}x^2$ $= \frac{2x(1-x)}{e^{2x}}$	2 marks correct answer 1 mark correct u' and v'

ii	$\cos^3(5x+3) = (\cos(5x+3))^3$ note $\cos^3(5x+3) \neq \cos(5x+3)^3$ $f(x) = \cos(5x+3)$ $f'(x) = -5\sin(5x+3)$ $y' = 3 \times -5\sin(5x+3)\cos^2(5x+3)$ $= -15\sin(5x+3)\cos^2(5x+3)$	2 marks correct solution 1 mark recognition of chain rule and attempt to derive
e	$\int \frac{3dx}{(3-2x)^3}$ this is not in $\frac{f'(x)}{f(x)}$ form $= 3\int (3-2x)^{-3}dx$ $= \frac{3}{-2 \times -2}(3-2x)^{-2} + c$ $= \frac{3}{4}(3-2x)^{-2} + c$	2 marks correct answer 1 mark an error in integration
f	$y = -\frac{1}{8}x^2 + x - 1$ $y + 1 = -\frac{1}{8}x^2 + x$ $-8(y+1) = x^2 - 8x$ $-8(y+1) + 16 = x^2 - 8x + 16$ $-8y + 8 = (x-4)^2$ $(x-4)^2 = -8(y-1)$ $\dot{V}(4, 1)$ $a = 2,\ a > 0$ concave down $\therefore S$ is (4,-1)	3 marks correct answer 2 marks correct vertex and focal length or cfe from vertex 1 mark correct vertex form

Question 12

a	$f'(x) = 2x - 5$ $2x - 5 = -3$ $2x = 2$ $x = 1$ $f(1) = 1 - 5 + 4$ $= 0$ point is $(1,0)$	2 marks for correct working and solution, also carry error if everything is correct. 1 mark for correct gradient.
b	$\frac{dy}{dx} = \sqrt{1-x^2} + x\left(-\frac{x}{\sqrt{1-x^2}}\right)$ $= \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}$ $= \frac{1-x^2-x^2}{\sqrt{1-x^2}}$ $= \frac{1-2x^2}{\sqrt{1-x^2}}$	3 marks for correct working and solution. 2 marks for 1 error. 1 mark for 2 errors.
c (i)	Point A $-2x = 3 - x$ $x = -3$ Point B $2x = 3 - x$ $x = 1$	1 mark for each x corrdinate
c (ii)	From the graph $x \geq 1$ $x \leq -3$	1 mark
d (i)	$y = \ln(\cos x)$ $\frac{dy}{dx} = \frac{-\sin x}{\cos x}$ $= -\tan x$	1 mark

d (ii)	$-\int_0^{\frac{\pi}{4}} \tan x \, dx$ $= -\ln\cos\frac{\pi}{4} + \ln\cos 0$ $= -\ln\frac{1}{\sqrt{2}} + \ln 1$ $= \ln\sqrt{2}$	2 marks for working and solution 1 mark for correct integral substitution.
e (i)	Range is 40ºC to 200ºC	1 mark
e (ii)	$\frac{2}{3} \times 120 = 80$ $80 = 120 + 80\sin\left(\frac{\pi}{3}t\right)$ $-0.5 = \sin\left(\frac{\pi}{3}t\right)$ $\frac{\pi}{3}t = \frac{7\pi}{6}$ $t = 3.5$ $\frac{dT}{dt} = \frac{80\pi}{3}\cos\left(\frac{\pi}{3}t\right)$ when $t = 3.5$ $\frac{dT}{dt} = \frac{80\pi}{3}\cos\left(\frac{\pi}{3} \times 3.5\right)$ $= \frac{80\pi}{3} \times -\frac{\sqrt{3}}{2}$ $= -\frac{40\sqrt{3}\,\pi}{3}$	3 marks for correct working and solution. 2 marks for correct derivative and t = 3.5 1 mark for either correct derivative or t = 3.5

Question 13

a	$\dfrac{1-\tan^2 x}{1+\tan^2 x} = 1 - 2s\sin^2 x$ $LHS = \dfrac{1-\tan^2 x}{\sec^2 x}$ $= \dfrac{1}{\sec^2 x} - \dfrac{\tan^2 x}{\sec^2 x}$ $= \cos^2 x - \left(\dfrac{\sin^2 x}{\cos^2 x} \times \left(\dfrac{1}{\sec^2 x}\right)\right)$ $= \cos^2 x - \left(\dfrac{\sin^2 x}{\cos^2 x} \times (\cos^2 x)\right)$ $= \cos^2 x - \sin^2 x$ $= (1-\sin^2 x) - \sin^2 x$ $= 1 - 2\sin^2 x$	3 marks- correct solution 2 marks- significant correct progress with one error ONLY 1 mark- correct use of one fundamental identity that leads to simplification
b	$f(x) = \dfrac{2x}{1+x^2}$ $A = \dfrac{1}{3}\left\{\dfrac{6}{10} + 1 + 4\left(\dfrac{4}{5} + 0\right) = 2(1)\right\}$ $= \dfrac{1}{3}\left\{\dfrac{8}{5} + \dfrac{16}{5} + 2\right\}$ $= \dfrac{1}{3}\left(\dfrac{34}{5}\right)$ $= \dfrac{34}{15}$ $= 2.27$	3 marks- correct solution 2 marks- significant correct progress with one error ONLY without QS 1 mark- correct *h* and function values

ci	$M(t) = Ae^{kt}$ $\frac{dM}{dt} = kAe^{kt}$ $= kM$ since $M = Ae^{kt}$	1 mark-correct answer
cii	$M(80) = Ae^{k(80)}$ $\frac{A}{2} = Ae^{80k}$ $\frac{1}{2} = e^{80k}$ $80k = \ln\left(\frac{1}{2}\right)$ $k = \ln\frac{\frac{1}{2}}{0}$ $= -0.008664...$ -0.00866	2 marks-correct answer 1 mark-ONLY one error in correct progress to value of k without QS 1 mark-ONLY error in sig figure rounding incorrect
ciii	$0.1A = Ae^{-0.0139t}$ $0.1 = e^{-0.008669t}$ $-0.00866t = \frac{\ln 0.1}{\ln e}$ $t = \frac{\ln(0.1)}{-0.00866}$ $= 265.75...$ $= 266hrs$	2 marks-correct answer 1 mark-ONLY one error in correct progress to value of t without QS
di	$2\cos x = \sec x$ $2\cos x = \frac{1}{\cos x}$ $2\cos^2 x = 1$ $\cos^2 x = \frac{1}{2}$ $\cos x = \pm\frac{1}{\sqrt{2}}$ $x = \frac{\pi}{4}$ (since $x > 0$ and first point $\int$)	1 mark-correct answer

dii	$V = \pi\int_0^{\frac{\pi}{4}} (2\cos x)^2 - (\sec x)^2\ dx$ $= \pi\int_0^{\frac{\pi}{4}} 4\cos^2 x - \sec^2 x\ \ dx$ $= \pi\left\{\frac{4}{2}\int_0^{\frac{\pi}{4}} \cos 2x + 1]^0, \frac{\pi}{4} - \int_0^{\frac{\pi}{4}} \sec^2 x\right\}$ $= \pi\left\{\ 2\left[\frac{1}{2}\sin 2x + x\right]^0, \frac{\pi}{4} - [\tan x]^0, \frac{\pi}{4}\right\}$ $= \pi\left\{[\sin 2x + 2x]^0, \frac{\pi}{4} - [\tan x]^0, \frac{\pi}{4}\right\}$ $= \pi\left\{\left[\ \sin 2\left(\frac{\pi}{4}\right) + \frac{2\pi}{4}\right) - (\sin 0 + 0)\right] - \left[\tan\frac{\pi}{4} - \tan 0\right]\right\}$ $= \pi\left\{\left[\ 1 + \frac{\pi}{2}\right] - 0 - [(1-0)]\right\}$ $= \pi\left\{\ 1 + \frac{\pi}{2} - 1\right\}$ $= \pi\left\{\frac{\pi}{2}\right\}$ $= \frac{\pi^2}{2}$	3 marks- correct solution 2 marks- significant correct progress with one error ONLY 1 mark- correct answer for sec2x from correct volume statement

Question 14

a) i)	$f(x) = x(x-2)^4$ Product rule $f'(x) = (x).4(x-2)^3 + (x-2)^4$ $= (x-2)^3(4x + (x-2))$ $= (x-2)^3(5x-2)$	1 mark correct solution to 2nd last line that shows factorising
ii)	If $(x-2)^3(5x-2) = 0$ $x = 2$ or $x = \frac{2}{5}$ at $x = 2$ $y = 0$ At $x = 0$ $f'(0) = 16$ at $x = \frac{2}{5}$ $f'\left(\frac{2}{5}\right) = 0$ at $x = 1$ $f'(1) = -3$ at $x = 2$ $f'(2) = 0$ and at $x = 3$ $f'(3) = 3$ $\therefore$ if $x < 2$ $f'(x) < 0$ and if $x > 2$ $f'(x) > 0$ hence a minimum at $x = \frac{2}{5}$ $y = 2.6$ $(1d.p.)$ if $x < \frac{2}{5}$ $f'(x) > 0$ and if $x > \frac{2}{5}$ $f'(x) < 0$ hence a maximum also at $x = 0$ $y = 0$	2 marks full correct solution 1 mark coordinates of both points or correct determination of both stationary points as max then min
iii)		2 marks correct solution or correct based on student answer 1 mark for either max or min graphed correct with correct shape and no further mistakes
b) i)	$y = f(x)$ is a reflection so is turned upside down and hence becomes $-\cos(2x)$ and because it is refected in the line $y = 1$ it moves up 2 units hence $+2$ $\therefore$ $f(x) = -\cos(2x) + 2$ $= 2 - \cos(2x)$	2 marks correct solution 1 mark for either negative cos or +2

b ii)	$\int_0^{2\pi} 2-\cos(2x)dx$ from the graph this is made up of 8 identical parts taken between 0 and $\frac{\pi}{2}$ taken below the curve and above the line $y = 1$ $Area = 8\int_0^{\frac{\pi}{2}} 2-\cos(2x) \; -1\, dx$ $= 8\int_0^{\frac{\pi}{2}} 1-\cos(2x)dx$ $= 8\left[x-\frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{2}}$ $= 8\left[\left(\frac{\pi}{2}-\frac{1}{2}\sin(\pi)\right)-0\right]$ $= 8\left[\frac{\pi}{2}-0\right]$ $= 4\pi\ units^2$ OR $\int_0^{2\pi} 2-\cos(2x) \quad -\cos(2x)\ dx$ $= \int_0^{2\pi} {}_2 - 2\cos(2x)\ dx$ $= \left[2x-\sin(2x)\right]_0^{2\pi}$ $= (2(2\pi)-\sin(4\pi))-(0-\sin(0))$ $= 4\pi\ units^2$	2 marks correct solution 1 mark correct applicant of areas between curves and satisfactory integration and substitution
c) i)	In ΔAXM and ΔBXC $\angle MAX = \angle BCX$ (alternate angles on parallel lines, AD and CB opposite sides of rectangle $ABCD$ given) $\angle AXM = \angle BXC$ (vertically opposite $\angle$) $\therefore \Delta AXM$ lll ΔBXC (equiangular)	2marks correct solution 1mark if did not give 2 correct reasons or no statement of test used for similarity

c ii)	$\frac{AM}{BC}=\frac{1}{2}$ (M is midpoint of AD (given)) and AD = BC (opp sides of rectangle $ABCD$) $\therefore \quad \frac{AX}{CX}=\frac{1}{2}$ (corresponding sides in similar Δ) $CX = 2AX$ now $AC = AX + CX$ $AC = AX + 2AX$ $AC = 3AX$ Show $3CX = 2AC$ $LHS = 3CX$ $= 3(2AX)$ $= 6AX$ $RHS = 2AC$ $= 2(3AX)$ $= 6AX$ LHS=RHS as required Or Using similar Δ as above $\frac{AX}{BC}=\frac{1}{2}$ $2AX = CX$ and $AX = AC - CX$ $\therefore \ 2(AC - CX) = CX$ $2AC - 2CX = CX$ $2AC = 3CX$ $\therefore \quad 3CX = 2AC$	1 mark correct solution Must logically show required statement
iii)	In ΔABC $\angle B$ is 90° $AC^2 = AB^2 + BC^2$ using $3CX = 2AC \therefore AC = \frac{3CX}{2}$ and $AB = 2AD$(*given*) and $BC = AD$ $\therefore BC = \frac{AB}{2}$ $\left(\frac{3CX}{2}\right)^2 = AB^2 + \left(\frac{AB}{2}\right)^2$ $\frac{9CX^2}{4} = AB^2 + \frac{AB^2}{4}$ $9CX^2 = 4AB^2 + AB^2$ $9CX^2 = 5AB^2$	2 marks correct solution 1mark correct substitution into Pythagoras or Correct use of part ii) and logical progress

Question 15

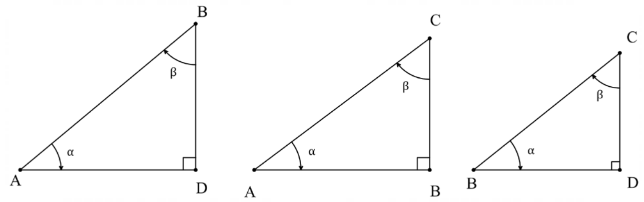
a	$\int_0^{\sqrt{e}} \frac{x^2}{x^3+e}\,dx$ $= \frac{1}{3}\int_0^{\sqrt{e}} \frac{3x^2}{x^3+e}\,dx$ $= \frac{1}{3}\left[\ln\lvert x^3+e\rvert\right]_0^{\sqrt{e}}$ $= \frac{1}{3}(\ln\lvert(\sqrt{e})^3+e\rvert - \ln\lvert 0^3+e\rvert)$ $= \frac{1}{3}(\ln(e\sqrt{e}+e) - \ln e)$ $= \frac{1}{3}\ln\left(\frac{e\sqrt{e}+e}{e}\right)$ $= \frac{1}{3}\ln(\sqrt{e}+1)$ $= \frac{\ln(\sqrt{e}+1)}{3}$	3 marks 1st mark for the correct integral 2nd mark for a correct substitution 3rd mark for correct solution using log laws
bi	6% p.a. = 0.5% per month On 1st December, the $4000 principal amount will have increased by 0.5% 11 times. $4000(1\cdot005)^{11} = \$4225\cdot58$	1 mark
bii	$A_0 = 4000$ $A_1 = 4000(1\cdot005) + M$ Note: For most of January, there is only $4000 in Jacqueline's account, so the bank rewards her with 0.5% interest on that amount (not $(4000+M)). $A_2 = A_1(1\cdot005) + M$ $= (4000(1\cdot005) + M) \times 1\cdot005 + M$ $= 4000(1\cdot005)^2 + M(1\cdot005) + M$ $= 4000(1\cdot005)^2 + M(1 + 1\cdot005)$ Keep going until a pattern can be seen. $A_3 = 4000(1\cdot005)^3 + M(1 + 1\cdot005 + 1\cdot005^2)$ $A_n = 4000(1\cdot005)^n + M(1 + 1\cdot005 + 1\cdot005^2 + \ldots + 1\cdot005^{n-1})$	3 marks correct solution (or correct given errors from part (a)) 2 marks for a minor error e.g. adding deposits before compounding interest or treating as 12 deposits (i.e. 31st Dec) 1 mark for some understanding, using geometric series

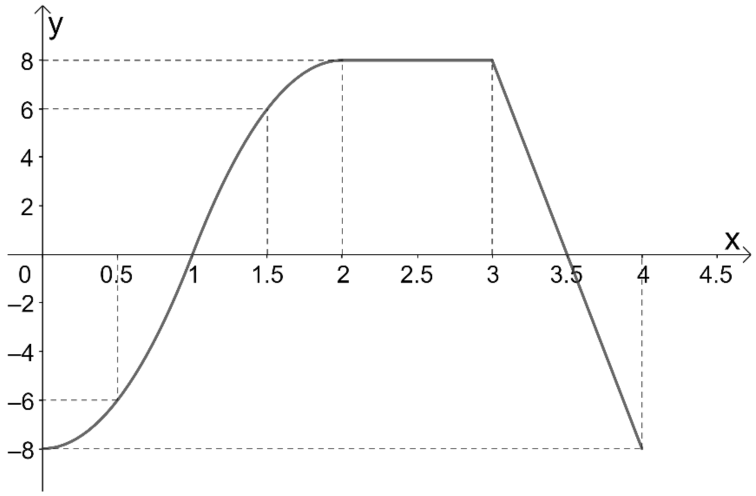
	On 1st December, there have been 11 deposits, and the value of the account is \$7000. $7000 = 4000(1{\cdot}005)^{11} + M(1 + 1{\cdot}005 + 1{\cdot}005^2 + ... + 1{\cdot}005^{10})$ The sum of the geometric series is: $S_{11} = \dfrac{1{\cdot}005^{11} - 1}{1{\cdot}005 - 1}$ $M \times S_{11} = 7000 - 4000(1{\cdot}005)^{11}$ $M = \dfrac{7000 - 4000(1{\cdot}005)^{11}}{S_{11}}$ $= \$245{\cdot}98$	
ci	C is one of three points of intersection of the cubic and the line. To find points of intersection: $4x^3 - 9x^2 = -5x$ $4x^3 - 9x^2 + 5x = 0$ $x(4x^2 - 9x + 5) = 0$ $x(4x - 5)(x - 1) = 0$ $x = 0, 1, \frac{5}{4}$ Note: From diagram, C is the middle point of intersection y-coord of C: $y = -5(1)$ $= -5$ $\therefore C(1, -5)$	2 marks 1 mark for solving the equations simultaneously Note: Several students mistook $x = \frac{5}{4}$ as the x-coordinate of C. Unfortunately this made part (iii) unprovable.
cii	$d = \dfrac{\lvert ax_1 + by_1 + c\rvert}{\sqrt{a^2 + b^2}}$ $(x_1, y_1) = A(x, y)$ Equation of line OC in general form: $y = -5x \Rightarrow 5x + y = 0$	2 marks (no explanation required for removing the absolute values) 1 mark for a correct expression using perpendicular distance formula with y=-5x and A(x,y)

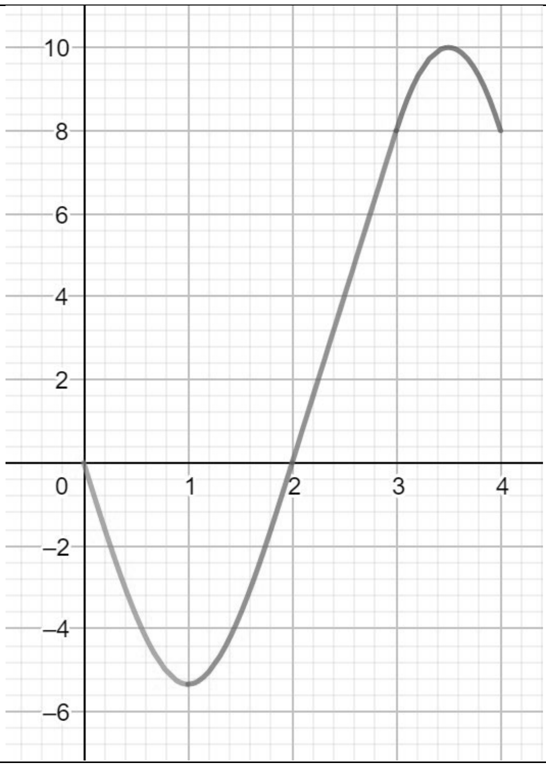
	$d = \dfrac{\lvert 5(x) + 5(y)\rvert}{\sqrt{5^2 + 1^2}}$ $= \dfrac{\lvert 5x + y\rvert}{\sqrt{26}}$ Since A lies on the cubic, the coordinates of A satisfy $y = 4x^3 - 9x^2$ $\therefore d = \dfrac{\lvert 5x + 4x^3 - 9x^2\rvert}{\sqrt{26}}$ $= \dfrac{\lvert 4x^3 - 9x^2 + 5x\rvert}{\sqrt{26}}$ Since the cubic is above the line in the domain $0 < x < 1$, $4x^3 - 9x^2 > -5x$ $4x^3 - 9x^2 + 5x$ is positive $\therefore d = \dfrac{4x^3 - 9x^2 + 5x}{\sqrt{26}}$ Note: What makes this question confusing for students is the non-numerical use of perpendicular distance formula, and the use of the pronumerals *x* and *y* as coordinates of point A, where *x* is between 0 and 1. It may be helpful to write x_A and y_A to denote the *x* and *y*-coordinates of point A.	There were several unsuccessful attempts to 'reverse-engineer' the solution from the result. The best method in this case is to simply use the perpendicular distance formula with point A and line OC to obtain an initial expression (1 mark), then try to obtain the result.
ciii	ΔAOC $A = \dfrac{1}{2}$ bh $= \dfrac{1}{2}(OC) \times d$ Pythagoras: $OC = \sqrt{1^2 + 5^2}$ $= \sqrt{26}$	1 mark

	$A = \frac{1}{2} \times \sqrt{26} \times \frac{1}{\sqrt{26}}(4x^3 - 9x^2 + 5x)$ $= \frac{1}{2}(4x^3 - 9x^2 + 5x)$ $= 2x^3 - \frac{9}{2}x^2 + \frac{5}{2}x$	
civ	$A = 2x^3 - \frac{9}{2}x^2 + \frac{5}{2}x$ $\frac{dA}{dx} = 6x^2 - 9x + \frac{5}{2}$ Stationary points: $\frac{dA}{dx} = 0$ $6x^2 - 9x + \frac{5}{2} = 0$ $12x^2 - 18x + 5 = 0$ $x = \frac{18 \pm \sqrt{(-18)^2 - 4(12)(5)}}{2(12)}$ $= \frac{18 \pm \sqrt{84}}{24}$ $= \frac{18 \pm 2\sqrt{21}}{24}$ $= \frac{9 \pm \sqrt{21}}{12}$ $= 0{\cdot}36811\ldots$ or $1{\cdot}131\ldots$ The *x*-coord of A is between 0 and 1 Therefore x = 0.36811 must be the value that produces a maximum area $A = 2(0{\cdot}36811)^3 - \frac{9}{2}(0{\cdot}36811)^2 + \frac{5}{2}(0{\cdot}36811)$ $= 0{\cdot}41$ square units (2dp)	3 marks 1st mark solving A'=0 2nd mark for choosing the correct value of x 3rd mark for substituting into A Note: Students did well attempting part (iv), even if they weren't able to prove the previous parts. Note: Normally we are required to determine the nature of a stationary point, but in this case we are told already that point A is chosen so that the area is a maximum, and since x=0.36811 is the only stationary point between 0 and 1, we know it must be the maximum.

Question 16

a-i	 $\Delta\ ABD \parallel\!\mid \Delta\ ABC$ $\therefore\ \frac{AB}{AC} = \frac{AD}{AB}$ $AB^2 = AC.AD$	1 mark – showing both first two lines. Identifying the similar triangles and then final statement was not sufficient.
a-ii	$\Delta\ ADB \parallel\!\mid \Delta\ BDC$ $\therefore\ \frac{BD}{CD} = \frac{AD}{BD}$ $BD^2 = CD.AD$ $AB^2 + BD^2 = AC.AD + CD.AD$ $= AD(AC + CD)$ $= AD(AD + CD + CD)$ $= AD(AD + 2CD)$ Pythagorean solution also accepted.	2 marks – full solution 1 mark – showing derivation of $AB^2 + BD^2 = AC.AD + CD.AD$ Pythagorean method – stating of a number of triads was not sufficient unless progress made towards solution.
b-i	Decreasing therefore $\frac{dy}{dx} < 0$ true for $0 < x < 1$ and $3.5 < x < 4$ also accepted $x < 1$ or $x > 3.5$ (should not really be using $\geq$ or $\leq$)	1 mark

b-ii	 $\int_0^2 f(x)dx = 0$ – given $\int_2^3 f(x)\ dx =$ a Area of Rectangle = 8 $\int_3^{3.5} f(x)dx = -\int_{3.5}^4 f(x)\ dx$ $\therefore \int_3^4 f(x)\ dx = 0$ therefore $f(4) = 0 + 8 + 0 = 8$	1 mark – correct answer
b-iii	Max occurs at $x = 3.5$ At $x = 2$, particle returns to origin. Distance from origin to the maximum equals area of trapezium. $A = \frac{8}{2}(1 + 1.5) = 10$	1 mark – correct answer

b-iv		2 marks correct answer 1 mark – correct answer with $f(0)$ and $f(2)$ correct at same displacement and one of $f(3.5)$ or $f(4)$
c-i	$P = 520\ 00$ $r = \frac{8.4}{12}\% = .7\% \Rightarrow R = 1.007$ M = monthtly repayment $A_1 = P \times R - M$ $A_2 = A_1 \times R - M = PR^2 - MR - M$ $A_3 = A_2 \times R - M = PR^3 - MR^2 - MR - M$ $A_3 = 520\ 000 \times 1.007^3 - M(1.007^2 + 1.007 + 1)$ $= 520\ 000 \times 1.007^3 - M\frac{1.007^3 - 1}{1.007 - 1}$	2 marks – derivation of correct formula – preference to have used the sum of geometric series 1 mark – correct formula without showing derivation.

c-ii	$n = 15 \times 12 = 180$ $A_{180} = 0$ $A_{180} = 520\,000 \times 1.007^{180} - M\frac{1.007^{180} - 1}{1.007 - 1}$ $M\frac{1.007^{180} - 1}{1.007 - 1} = 520\,000 \times 1.007^{180}$ $M = 520\,000 \times 1.007^{180} \times \frac{1.007 - 1}{(1.007^{180} - 1)}$ $M = 5090.2104$ $M = 5090.21$	2 marks - correct solution 1 mark – correct expression for $A_{180}=0$
c-ii	$A_{24} = 520\,000 \times .007^{24} - 5090.21\frac{(1.007^{24} - 1)}{1.007 - 1}$ $= 482243.89$ Amount owing = 482243.89-20000=462243.89 $An = 462243.89 \times 1.007^{n} - 5090.21\frac{(1.007^{n} - 1)}{1.007 - 1}$ $0 = 462243.89 \times 1.007^{n} - 5090.21((1.007^{n} - 1)) \times \frac{1000}{7}$ $0 = \frac{7}{1000} \times 462243.89 \times 1.007^{n} - 5090.21((1.007^{n} - 1))$ $-5090.21 = \left(\frac{7}{1000} \times 462243.89 - 5090.21\right)(1.007^{n})$ $1.007^{n} = \frac{-5090.210}{\frac{7}{1000} \times 462243.89 - 5090.21} = 2.744784$ $n(\log 1.007) = \log 2.744784$ $n = \frac{\log 2.744784}{\log 1.007} = 144.74747 \cong 145$	3 marks – correct answer 2 marks – show single term with n 1 mark – new amount owing